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Supergravity on $AdS_{4/7} \times S^{7/4}$ and M branes

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Abstract: We calculate the dimensions of operators in three and six dimensional superconformal field theories by using the duality between these theories at large N and $D = 11$ supergravity on $AdS_{4/7} \times S^{7/4}$. We find that for the duality relations to work the Kaluza-Klein masses given in the supergravity literature must be rescaled and/or shifted.

Keywords: String Duality, Conformal Field Models in String Theory.

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Contents

1. Introduction

Recently a duality between some superconformal brane world-volume theories and supergravity on corresponding AdS spaces was conjectured in ref. [1]. More precisely it was argued that for D3 branes in a certain limit near the branes the large N world-volume dynamics is dual to IIB supergravity on $AdS_5 \times S^5$. This background is precisely the near horizon geometry of the D3 branes and one can probe it by supergravity because the radius of the AdS space and the sphere is large (proportional to a positive power of N times the Planck length) or the curvature is small. The geometry near the D3 branes is also given by the S^5 compactification of IIB supergravity which is maximally supersymmetric, i.e. gauged supergravity. Using this duality the isometry of the AdS_5 space $SO(3,2)$ becomes the conformal symmetry on the brane theory whereas the isometry of S^5 which is $SO(6)$ becomes the R symmetry. Results about entropy of and scattering from black holes made of D3 branes seem to support this conjecture [2, 3, 4, 5, 6]. The conjecture can also be applied to other non-dilatonic branes, i.e. the membranes and fivebranes of $D = 11$ supergravity (or M theory). In that case the duality is between large N world-volume theories of M2/M5 branes and $D = 11$ supergravity (M theory) on $AdS_{4/7} \times S^{7/4}$. Recently other related work appeared in [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

A much more precise formulation of this duality was given in ref. [9]. There it was shown that supergravity on an AdS_{d+1} background is equivalent to a superconformal field theory (SCFT) on the boundary of the AdS_{d+1} space which is d dimensional Minkowski space M_d. To every supergravity field on AdS_{d+1} there is a corresponding chiral operator in the SCFT on the boundary. For scalars, if the field is massive, massless or tachyonic the operator in the SCFT is irrelevant, marginal and relevant

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respectively. Moreover, the dimension of the operator is fixed by the mass of the field in supergravity. As a result this duality can be checked by comparing the spectrum of supergravity on AdS_{d+1} backgrounds and dimensions of the operators in the corresponding SCFT living on M_d. The spectrum of IIB supergravity on $AdS_5 \times S^5$ is known [29] and matches precisely the dimensions of operators of $\mathcal{N}=4$ $D=4$ super Yang-Mills theory which is the world-volume theory of D3 branes [9].

In this paper we repeat this for the dualities between large N world-volume theories of M2/M5 branes and supergravity on $AdS_{4/7} \times S^{7/4}$. [1]. Section 2 is a short review of the results of ref. [9] that we need and their relation to previous results in the supergravity literature. In sections 3 and 4 we calculate the dimensions of the relevant and marginal operators in $D=3$ and $D=6$ SCFT with sixteen supercharges using the spectrum of the $D=11$ supergravity compactified on S^7 and S^4 respectively. Section 5 is a short discussion of our results.

2. Supergravity on AdS_{d+1}

In ref. [9] it was shown that supergravity on AdS_{d+1} is equivalent to a SCFT on the boundary M_d. The isometry group $SO(d,2)$ of the AdS_{d+1} space is also the conformal symmetry of the d dimensional boundary SCFT. For a scalar field ϕ which has a value ϕ_0 on the boundary there is a coupling of the form $\int \phi_0 \mathcal{O}$ to an operator $\mathcal{O}$ on the boundary M_d. The conformal dimension of $\mathcal{O}$ is related to the mass of ϕ by

$$\tilde{m}^2 = \Delta(\Delta - d).$$

This shows that scalar fields which are massive, massless and tachyonic correspond to irrelevant, marginal and relevant operators on the boundary. For a p form field in supergravity the relation becomes

$$\tilde{m}^2 = (\Delta + p)(\Delta + p - d). \tag{2}$$

Since we will use these formulas and the spectrum of $D = 11$ supergravity on $AdS_{4/7} \times S^{7/4}$ to obtain the dimensions of operators $\mathcal{O}$ it is important to review also the supergravity conventions. Supergravity on AdS_{d+1} has the isometry group $SO(d,2)$ which is the conformal group of the boundary theory. This has a maximal subgroup of $SO(d) \times SO(2)$. The eigenvalues of the $SO(2)$ factor are denoted by E_0 and give the dimension of operators in the SCFT on the boundary [13]. These operators are bilinear currents of the boundary theory which correspond to the supergravity fields in the bulk. For example, in the AdS_5 case one has for scalars

$$m^2 = E_0(E_0 - 4), \tag{3}$$

which is exactly eq. (2). Therefore in this case $E_0 = \Delta$ and $\tilde{m}^2 = m^2$. (Throughout the paper $\tilde{m}^2$ refers to the mass which appears in the definition of Δ whereas m^2 refers

[1]While this work was being completed [28], which has the same results, appeared.

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to the mass which appears in the supergravity literature and enters in the definition fo E_0.) We will see that this is not always the case for E_0 which appear in the supergravity literature, i.e. in some cases $E_0 \neq \Delta$ if one identifies $\tilde{m}^2$ with m^2. However, both E_0 and Δ give the conformal dimension of an operator $\mathcal{O}$ so they must be equal. Requiring this equality gives a relation between m^2 and $\tilde{m}^2$, it turns out that the spectrum of spherically compactified supergravity which appears in the literature must be rescaled and/or shifted in these cases to make contact with ref. [9]. In addition, the formula for p forms must be modified. This is because in supergravity the mass is related to a Maxwell operator whereas in [9] it is related to a Laplacian which is different. For example, in the AdS_5 case for one forms the supergravity result is

$$m^2 = (E_0 - 1)(E_0 - 3). \tag{4}$$

Therefore eq. (2) must be changed to

$$\tilde{m}^2 = (\Delta - p)(\Delta + p - d).$$

3. $D = 11$ supergravity on $AdS_4 \times S^7$ and M2 branes

In this section we consider $D = 11$ supergravity on $AdS_4 \times S^7$ and its dual large N world-volume theory of M2 branes which is a $D = 3$ $\mathcal{N} = 8$ SCFT with sixteen supercharges. This is the world-volume theory of N M2 branes or equivalently the world-volume theory of N D2 branes at infinitely strong coupling. In contrast to those of N D2 branes (at finite coupling) the degrees of freedom of N M2 branes are not known. The isometry group of AdS_4 is $SO(3,2)$ which is also the conformal group of the M2 brane theory. The isometry group of S^7 which is $SO(8)$ becomes the R symmetry of the boundary theory.

The spectrum of $D = 11$ supergravity on $AdS_4 \times S^7$ is given in ref. [30, 31, 32]. There are nine towers of KK states: five scalars, three vectors and a symmetric tensor which includes the graviton. The five dimensional supergravity multiplet is given by a scalar, a pseudoscalar and a vector in addition to the graviton. These have supergravity masses

$$m_{0^+}^2 = (k-3)^2 - 1, \qquad m_{0^-}^2 = k^2 - 1, \qquad m_1^2 = k^2 - 1, \tag{6}$$

where $k \geq 2$ for the scalar and $k \geq 1$ for the other two cases. These are the only KK towers which include massless and tachyonic fields which correspond to the only marginal and relevant operators of the three dimensional SCFT on the boundary.

However, as mentioned above before these expressions for the masses can be used one has to impose $E_0 = \Delta$. In this case for all bosons [32]

$$E_0 = \frac{3}{2} + \frac{1}{2}\sqrt{1 + m^2}. \tag{7}$$

This has to be equal to

$$\Delta = \frac{3}{2} + \frac{1}{2}\sqrt{9 + \tilde{m}^2}. \tag{8}$$

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We find that the supergravity masses for the scalars must be transformed to

$$\tilde{m}^2_{0+} = \frac{1}{4}(m^2 - 8) \qquad (9),$$

so that the eq. (2) for Δ can be applied. Note that the masses are both rescaled and shifted in this case. The scalar masses become

$$\tilde{m}^2_{0+} = \frac{1}{4}(k-4)(k-2) - 2, \qquad k \geq 2. \qquad (10)$$

This corresponds to operators with dimension $\Delta = k/2$. We find that there are relevant operators of dimension $1, 3/2, 2, 5/2$ and a marginal operator of dimension 3. These operators transform as the 35 representation of the R symmetry group $SO(8)$ which is the symmetric traceless representation. Similarly the pseudoscalar masses become

$$\tilde{m}^2_{0-} = \frac{1}{4}(k^2 - 1) - 2, \qquad k \geq 1. \qquad (11)$$

This corresponds to operators with $\Delta = 3/2 + k/2$. In this case there are two relevant operators of dimension $2, 5/2$ and one marginal operator. These are also in the 35 of $SO(8)$. For the vectors using eq. (2) with $p = 1$ we find

$$\Delta = \frac{3}{2} + \frac{1}{2}\sqrt{1 + \tilde{m}^2}. \qquad (12)$$

Comparing this with E_0 we get $\tilde{m}^2 = m^2/4$ so that

$$\tilde{m}^2_1 = \frac{1}{4}(k^2 - 1) - 2, \qquad k \geq 1. \qquad (13)$$

Note that in this case there is no shift. This leads to the dimensions $\Delta = 3/2 + k/2$ for the operators which couple to vectors. In this case the operators are in the 28 of $SO(8)$ which is the antisymmetric representation.

Above we found the dimensions and the R charges of the marginal and relevant operators in the $D = 3$ SCFT with sixteen supercharges at alrge$N$ using the duality conjectured in [1] and the relations given in [9]. This was done without the knowledge of the fundamental degrees of freedom of the world-volume theory of N M2 branes. (However in [34] it was argued that these operators can be built from scalars and fermions living on a the world-volume of N M2 branes.)

4. $D = 11$ supergravity on $AdS_7 \times S^4$ and M5 branes

In this section we consider $D = 11$ supergravity on $AdS_7 \times S^4$. The dual theory is the large N limit of M5 brane world-volume theory which is the non-Abelian tensor theory with $(0, 2)$ supersymmetry. This can also be seen as the world-volume theory of N NS5 branes at infinite coupling. In this case we know the degrees of freedom of niether N NS5 nor M5 brane world-volume theories. The isometry groups of AdS_7 and

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S^4 which are $SO(6,2)$ and $SO(5)$ are the conformal and R symmetries of the $D=6$ world-volume theory.

The spectrum of $D=11$ supergravity on $AdS_7 \times S^4$ is given in ref. [35, 36, 37]. There are seven towers of bosonic KK states: two scalars, two vectors, two two forms and a symmetric tensor tower which includes the graviton. Among these only two, one scalar and one vector KK tower can have massless and/or tachyonic states. Consider first the scalars with supergravity masses

$$m^2 = k(k-3), \qquad k \geq 2. \tag{14}$$

E_0 in this case is given by [36]

$$E_0 = 3 + \frac{1}{2}, \sqrt{36 + 16m^2}, \tag{15}$$

whereas from eq. (2) we get

$$\Delta = 3 + \frac{1}{2}\sqrt{36 + 4\tilde{m}^2}. \tag{16}$$

We find that we need to rescale the supergravity masses, $\tilde{m}^2 = 4m^2$ so that now

$$\tilde{m}_0^2 = 4k(k-3), \qquad k \geq 2. \tag{17}$$

Using this we find for the dimension of the operators $\Delta = 2k$. Thus there is one relevant operator of dimension 4 and one marginal operator coming from the scalars. For the vectors the supergravity masses are

$$m^2 = k^2 - 1, \qquad k \geq 1. \tag{18}$$

E_0 is given by (this includes an extra shift which is required to have a well-defined number operator)

$$E_0 = 3 + \frac{1}{2}\sqrt{16 + 16m^2}. \tag{19}$$

Using eq. (2) for Δ for a one form we find that again $\tilde{m}^2 = 4m^2$ so that

$$\tilde{m}_1^2 = 4(k^2 - 1), \qquad k \geq 1, \tag{20}$$

which gives $\Delta = 3+2k$. We find that the vectors give one relevant operator of dimension 5. We found the dimensions of the marginal and relevant operators of the $D=6$ SCFT with sixteen supercharges at large N using the duality of [1] and [9]. As in the $D=3$ case we do not know the fundamental degrees of freedom on the brane world-volume of N M5 branes (however see [34] again for a possible description of these).

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5. Conclusions

In this paper we calculated the dimensions of marginal and relevant operators in three and six dimensional SCFT with sixteen supercharges. This was done by using the KK spectrum of $D = 11$ supergravity compactified on S^4 and S^7 and using the results of [9]. We found that if the conformal weight of operators Δ in [9] is identified with the dimension of the boundary operators E_0 in the supergravity literature one finds that the KK masses m^2 and $\tilde{m}^2$ in the two cases are not equal. In particular the masses in the supergravity literature must be rescaled and/or shifted in order to be used in relations given in [9]. The rescaling of the masses can be explained as follows.[2] In ref. [9] the radius of the AdS_{d+1} spacetime and the sphere were taken to be the same for all d. On the other hand, in supergravity this is only true for AdS_5; for the other cases we investigated in this paper one has $R_{AdS_7} = 2R_{S^4}$ and $R_{AdS_4} = R_{S^7}/2$. As a result, the supergravity masses must be rescaled (by factors of 4 and 1/4) before formulas of ref. [9] can be used. The shift in the scalar masses for the AdS_4 case is due to the conformal mass factor for the scalars in four dimensional gravity which gives $m^2 \to m^2 + R/6$.

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References

[1] J.M. Maldacena, `hep-th/9711200`.

[2] S.S. Gubser, I.R. Klebanov and A.W. Peet.

[3] I.R. Klebanov, *Nucl. Phys.* **B 496** (1997) 231 [`hep-th/9702076`].

[4] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *Nucl. Phys.* **B 499** (1997) 217 [`hep-th/9703040`].

[5] S.S. Gubser and I.R. Klebanov, *Phys. Lett.* **B 413** (1997) 41 [`hep-th/9708005`].

[6] J.M. Maldacena and A. Strominger, `hep-th/9710014`.

[7] A.M. Polyakov, `hep-th/9711002`.

[8] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, `hep-th/9802109`.

[9] E. Witten, `hep-th/9802150`.

[10] K. Sfetsos and K. Skenderis, `hep-th/9711138`.

[2] This was pointed out to me by A. Brandhuber and N. Itzhaki and also by R. Kallosh.

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[11] R. Kallosh, J. Kumar and A. Rajaraman, hep-th/9712073

[12] P. Claus, R. Kallosh, J. Kumar, P. Townsend and A. Van Proeyen, hep-th/9801206.

[13] S. Ferrara and C. Fronsdal, hep-th/9712239.

[14] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, hep-th/9802042.

[15] M. Gunaydin and D. Minic, hep-th/9802047.

[16] G.T. Horowitz and H. Ooguri, hep-th/9802116.

[17] S. Kachru and E. Silverstein, hep-th/9802183.

[18] M. Berkooz, hep-th/9802195.

[19] V. Balasubramanian and F. Larsen, hep-th/9802198.

[20] S.J. Rey and J. Yee, hep-th/9803001.

[21] J.M. Maldacena, hep-th/9803002.

[22] M. Flato and C. Fronsdal, hep-th/9803013.

[23] A. Lawrence, N. Nekrasov and C. Vafa, hep-th/9803015.

[24] S.S. Gubser, A. Hashimoto, I.R. Klebanov and M. Krasnitz, hep-th/9803023.

[25] I.Ya. Aref'eva and I.V. Volovich, hep-th/9803028.

[26] L. Castellani, A. Ceresole, R. D'Auria, S. Ferrara, P. Fre' and M. Trgiante, hep-th/9803039.

[27] S. Ferrara, C. Fronsdal and A. Zaffaroni, hep-th/9802203.

[28] O. Aharony, Y. Oz and Z. Yin, hep-th/9803051.

[29] H.J. Kim, L. J. Romans and P. van Nieuwenhuizen, *Phys. Rev.* **D 32** (1985) 389.

[30] B. Biran, A. Casher, F. Englert, M. Rooman and P. Spindel, *Phys. Lett.* **B 134** (1984) 179.

[31] M. Gunaydin and N. P. Warner, *Nucl. Phys.* **B 272** (1986) 99.

[32] A. Casher, F. Englert, H. Nicolai and M. Rooman, *Nucl. Phys.* **B 243** (1984) 173.

[33] P. Van Nieuwenhuizen, *Class. and Quant. Grav.* **2** (1985) 1.

[34] S Minwalla, hep-th/9803053.

[35] L. Castellani, R. D'Auria, P. Fre, K. Pilch and P. Van Nieuwenhuizen, *Class. and Quant. Grav.* **1** (1984) 339.

[36] M. Gunaydin, P. van Nieuwenhuizen and N.P. Warner, *Nucl. Phys.* **B 255** (1985) 63.

[37] K. Pilch, P. van Nieuwenhuizen and P.K. Townsend, *Nucl. Phys.* **B 242** (1984) 377.